CoClust: A Python Package for Co-clustering

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Plan

From Document Clustering...

- Document clustering techniques are widely used in text mining applications.
- When using a document-term matrix, the goal is to group rows (documents) into different clusters so that documents assigned to a given cluster are more similar to each other than to those in other clusters.

... to Document Co-clustering

- Document co-clustering is a natural extension of standard clustering where the rows (documents) and columns (words) of a document-term matrix are simultaneously grouped into meaningful blocks called co-clusters
- Co-clustering makes large document sets easier to handle and interpret !

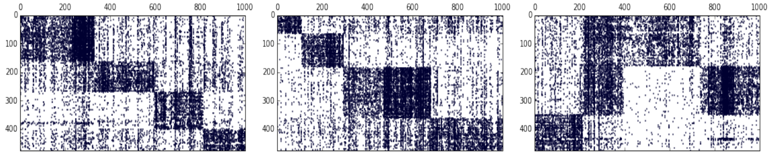


Figure –

Poor Availability of Text Co-clustering Tools in the Python Community

- In contrast to clustering, not so many packaged Python implementations are available for document co-clustering.
  - A few co-clustering implementations are available in **scikit-learn**, but they are mostly limited to spectral methods.
- The goal of the package presented here is to give access to a larger range of methods for co-clustering textual documents.

Plan

Notation

Data

- matrix $\mathbf{X} = (x_{ij})$
- $i \in I$ set of n documents, $j \in J$ set of d terms

Partition of I in g clusters

- $\mathbf{Z} = (z_{ik})$ where $z_{ik} = 1$ if $i \in k$th cluster and $z_{ik} = 0$ otherwise

z	Z		
3	0	0	1
2	0	1	0
3	0	0	1
2	0	1	0
1	1	0	0

Partition of J in s clusters

- $\mathbf{W} = (w_{ij})$ where $w_{j\ell} = 1$ if $j \in \ell$th cluster and $w_{j\ell} = 0$ otherwise

From $\mathbf{Z}$ and $\mathbf{W}$

- Block (k, ℓ) is defined by the x_{ij}'s with $z_{ik} w_{j\ell} = 1$

High-level View of Available Methods

Labiod & Nadif 2011 ; Ailem, Role, Nadif, 2015

- Modularity-based methods
- **CoclustSpecMod** and **CoclustMod**

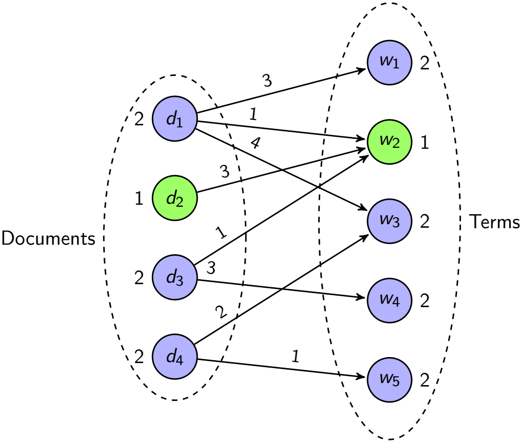
Govaert & Nadif, 2013

- Information theoretic based methods
- **CoclustInfo**

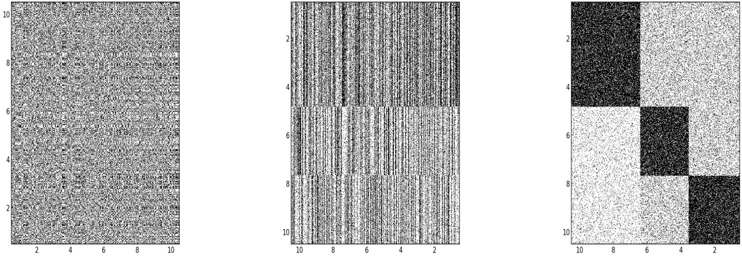
Govaert & Nadif, 2013 ; Ailem, Role, Nadif 2015, 2016

- Model-based, probabilistic methods soon to be included

Bipartite, term-document graph



Diagonal co-clustering



Standard Modularity

Find a partition of the nodes that maximizes :

$$\frac{1}{2|E|}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{g}(a_{ij}-\frac{a_{i.}a_{j.}}{2|E|})z_{ik}z_{jk} \qquad (1)$$

where $|E|$ is the total number of edges, $a_{i.}=\sum_{i'}a_{ii'}$ is the degree of node i and $z_{ik}=1$ if node i belongs to cluster k and 0 else.

Bipartite Modularity

Find a partition that maximizes :

$$\frac{1}{a_{..}}\sum_{i=1}^{n}\sum_{j=1}^{d}\sum_{k=1}^{g}(a_{ij}-\frac{a_{i.}a_{.j}}{a_{..}})z_{ik}w_{jk} \tag{2}$$

where $a_{..}$ is the total number of edges, $a_{i.}=\sum_{i'}a_{ii'}$ is the degree of node i and $Z=(z_{ik})$ and $W=(w_{jk})$ are the binary indicator matrices for the rows and columns resp.

CoclustSpecMod

- **CoclustSpecMod** uses an approach close to the spectral co-clustering algorithm proposed by Dhillon (Dhillon 2001), except it relies on a spectral approximation of the modularity matrix. The main steps are as follows :
  - form a low-dimensional embedding of the data into an Euclidean space.
  - perform an hard-assignment (e.g. k-means) on this new space to obtain a simultaneous clustering of the row and columns.

CoclustMod : direct, alternated maximization of modularity

Given current Z and W

1) for each row i, since :

$$\frac{1}{a_{..}}\sum_{i=1}^{n}\sum_{j=1}^{d}\sum_{k=1}^{g}(a_{ij}-\frac{a_{i.}a_{.j}}{a_{..}})z_{ik}w_{jk}=\frac{1}{a_{..}}\sum_{i=1}^{n}\sum_{k=1}^{g}\left(a_{ik}^{W}-\frac{a_{i.}a_{.k}^{W}}{a_{..}}\right)z_{ik}$$

assign the row to the cluster k maximizing $\left(a_{ik}^{W}-\frac{a_{i.}a_{.k}^{W}}{a_{..}}\right)$.

2) Then, for each column j, since :

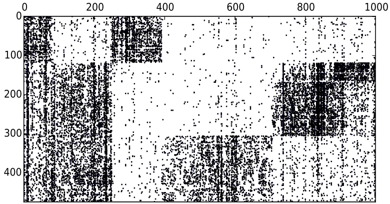
$$\frac{1}{a_{..}}\sum_{i=1}^{n}\sum_{j=1}^{d}\sum_{k=1}^{g}(a_{ij}-\frac{a_{i.}a_{.j}}{a_{..}})z_{ik}w_{jk}=\frac{1}{a_{..}}\sum_{i=1}^{d}\sum_{k=1}^{g}\left(a_{kj}^{Z}-\frac{a_{.j}a_{k.}^{Z}}{a_{..}}\right)w_{jk}$$

assign the column to the cluster k maximizing $\left(a_{kj}^{Z}-\frac{a_{.j}a_{k.}^{Z}}{a_{..}}\right)$

where $A^{W} := \{a_{ik}=\sum_{j=1}^{d} w_{jk}a_{ij}; i=1,\ldots,n; k=1,\ldots,g\}$ and $A^{Z} := \{a_{kj}=\sum_{i=1}^{n} z_{ik}a_{ij}; j=1,\ldots,d; k=1,\ldots,g\}$

Non-diagonal Co-clustering

- Sometimes, you don't seek a diagonal structure.
- **CoclustInfo** is a valuable method for addressing this situation.



From an Initial Joint Distribution...

Given two variables I and J taking values in the sets $I = \{1, \ldots, i, \ldots, n\}$ of rows and $J = \{1, \ldots, j, \ldots, d\}$ of columns respectively, compute their associated joint distribution P_{IJ}.

	1	2	3	4	5	
1	5	4	6	1	0	16
2	6	5	4	0	1	16
3	1	0	1	7	5	14
4	1	1	0	6	5	13
5	4	5	3	4	5	21
6	5	4	4	3	4	20
	22	19	18	21	20	100

	1	2	3	4	5	
1	0.05	0.04	0.06	0.01	0.00	0.16
2	0.06	0.05	0.04	0.00	0.01	0.16
3	0.01	0.00	0.01	0.07	0.05	0.14
4	0.01	0.01	0.00	0.06	0.05	0.13
5	0.04	0.05	0.03	0.04	0.05	0.21
6	0.05	0.04	0.04	0.03	0.04	0.20
	0.22	0.19	0.18	0.21	0.20	1.00

...Define a new "aggregated" Joint Distribution

- Let **z** and **w** be partitions into g clusters and m clusters of the set I of the rows and the set J of columns of $X = (x_{ij})$
- Define two new random variables K and L taking values in the sets $K = \{1, \ldots, g\}$ and $L = \{1, \ldots, m\}$
- Define a new table $X^{\mathbf{zw}} = (x^{\mathbf{zw}}_{k\ell})$ according to the partitions **z** and **w** :

$$x^{\mathbf{zw}}_{k\ell} = \sum_{i,j} z_{ik} w_{j\ell} x_{ij} \qquad \forall k \in K \quad \text{and} \quad \forall \ell \in L.$$

- $p^{\mathbf{zw}}_{k\ell} = \frac{x^{\mathbf{zw}}_{k\ell}}{N} = \sum_{i,j} z_{ik} w_{j\ell}\ p_{ij}$

CoclustInfo : Minimize the loss in mutual information

- Seek partitions $\mathbf{z}$ and $\mathbf{w}$ that minimize the loss in mutual information between the two distributions

$$\mathcal{I}(P_{IJ}) = \sum_{i,j} p_{i,j} \log \frac{p_{i,j}}{p_i p_j}$$

$$\mathcal{I}(P_{KL}^{\mathbf{zw}}) = \sum_{k,\ell} p_{k\ell}^{\mathbf{zw}} \log \frac{p_{k\ell}^{\mathbf{zw}}}{p_{k.}^{\mathbf{z}} p_{.\ell}^{\mathbf{w}}}$$

$$\mathcal{I}(P_{IJ}) - \mathcal{I}(P_{KL}^{\mathbf{zw}})$$

Plan

Coclust offers

- several effective algorithms for co-clustering based on different approaches
- easy-to-use tools for the interpretation of co-clusters
- easy-to-use tools for comparing the obtained results

Work in progress

- Integration of different algorithms based on the latent block models
- New tools for interpretation

Bibliography

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